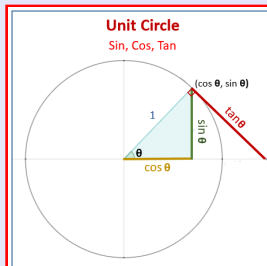


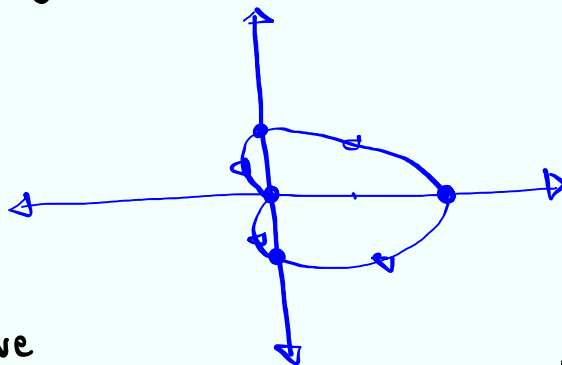
# Trigonometry Final Exam Review



Feb 19-8:47 AM

Graph  $r = 1 + \cos \theta$

$\theta$	$r$
$0^\circ$	2
$90^\circ$	1
$180^\circ$	0
$270^\circ$	1
$360^\circ$	2



Solve

$$r=0$$

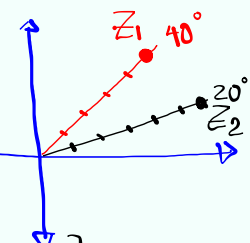
$$1 + \cos \theta = 0$$

$$\cos \theta = -1$$

$$\theta = 180^\circ$$

Dec 9-10:33 AM

$$Z_1 = 5(\cos 40^\circ + i \sin 40^\circ)$$

$$Z_2 = 6(\cos 20^\circ + i \sin 20^\circ)$$


$$Z_1 \cdot Z_2 = 5 \cdot 6 [\cos(40^\circ + 20^\circ) + i \sin(40^\circ + 20^\circ)]$$

$$= 30 [\cos 60^\circ + i \sin 60^\circ]$$

$$\frac{Z_1}{Z_2} = \frac{5}{6} [\cos(40^\circ - 20^\circ) + i \sin(40^\circ - 20^\circ)]$$

$$= \frac{5}{6} [\cos 20^\circ + i \sin 20^\circ]$$

$$Z_1^3 = 5^3 [\cos(3 \cdot 40^\circ) + i \sin(3 \cdot 40^\circ)]$$

$$= 125 [\cos 120^\circ + i \sin 120^\circ]$$

Dec 9-10:36 AM

find all cube roots of  $8 \text{cis } 60^\circ$

3 Ans.  
 $n=3$   
 $k=0, 1, 2$

$$8 [\cos 60^\circ + i \sin 60^\circ]$$

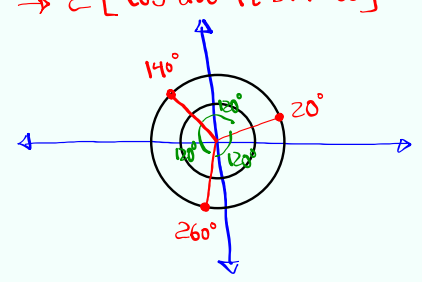
$$\sqrt[3]{8} \left[ \cos \frac{60^\circ + k \cdot 360^\circ}{3} + i \sin \frac{60^\circ + k \cdot 360^\circ}{3} \right]$$

$$= 2 [\cos(20^\circ + k \cdot 120^\circ) + i \sin(20^\circ + k \cdot 120^\circ)]$$

$k=0 \rightarrow 2 [\cos 20^\circ + i \sin 20^\circ]$

$k=1 \rightarrow 2 [\cos 140^\circ + i \sin 140^\circ]$

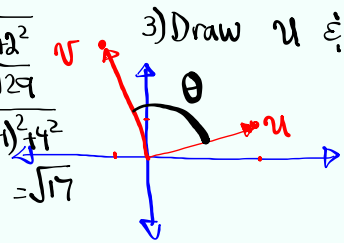
$k=2 \rightarrow 2 [\cos 260^\circ + i \sin 260^\circ]$



Dec 9-10:42 AM

$u = \langle 5, 2 \rangle$     $|u| = \sqrt{5^2 + 2^2} = \sqrt{29}$   
 $v = \langle -1, 4 \rangle$     $|v| = \sqrt{(-1)^2 + 4^2} = \sqrt{17}$

3) Draw  $u$  &  $v$



1)  $u + v = \langle 4, 6 \rangle$

2)  $u - v = \langle 6, -2 \rangle$

4)  $u \cdot v = 5(-1) + 2(4) = -5 + 8 = 3$

5) Find the angle between them.

$$\cos \theta = \frac{u \cdot v}{|u||v|}$$

$$\cos \theta = \frac{3}{\sqrt{29} \cdot \sqrt{17}}$$

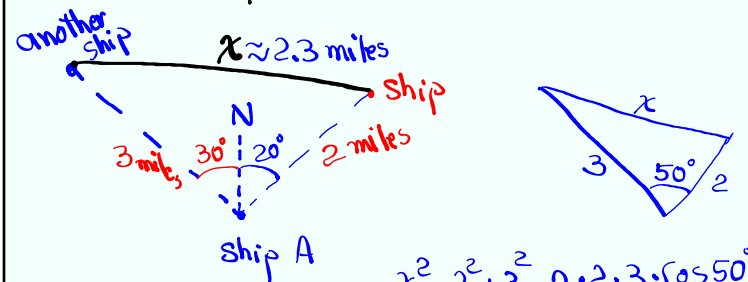
$$\cos \theta \approx .135$$

$$\theta = \cos^{-1}(.135)$$

$$\theta \approx 82^\circ$$

Dec 9-10:48 AM

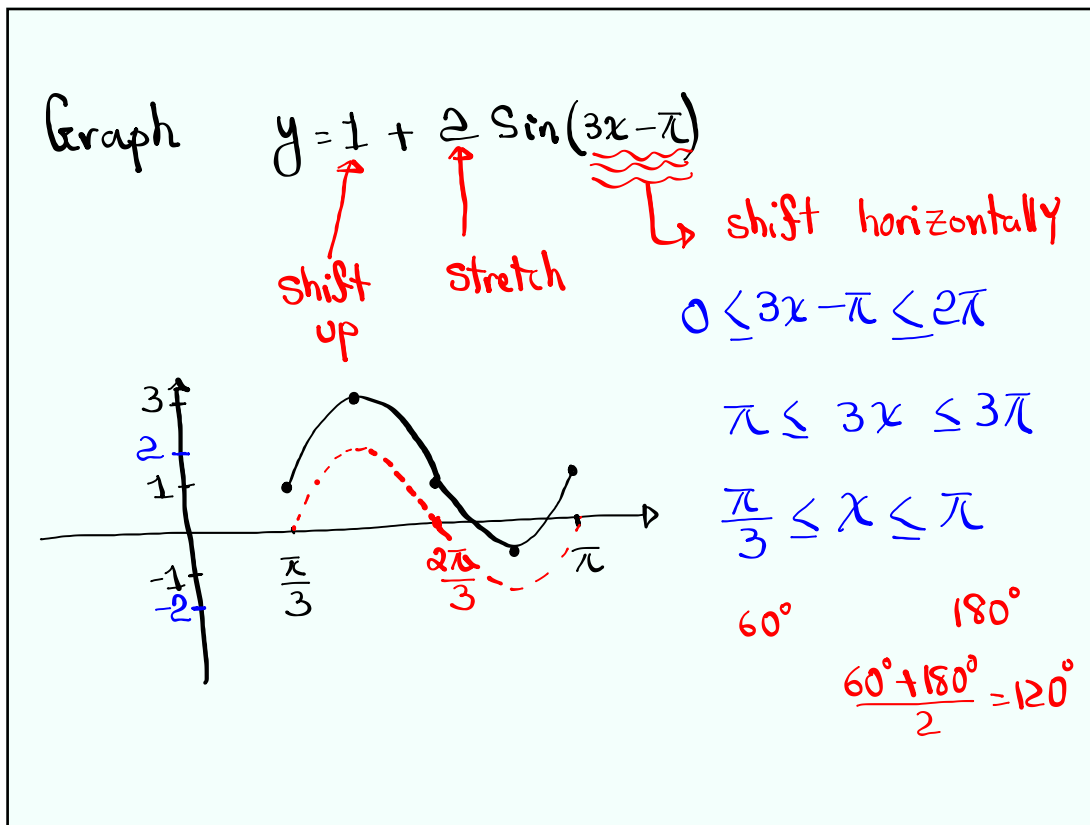
ship A is 2 miles from another ship with bearing  $N 20^\circ E$ .  
 ship A is 3 miles from a third ship with bearing  $S 30^\circ W$ .  
 How far apart are these ships?



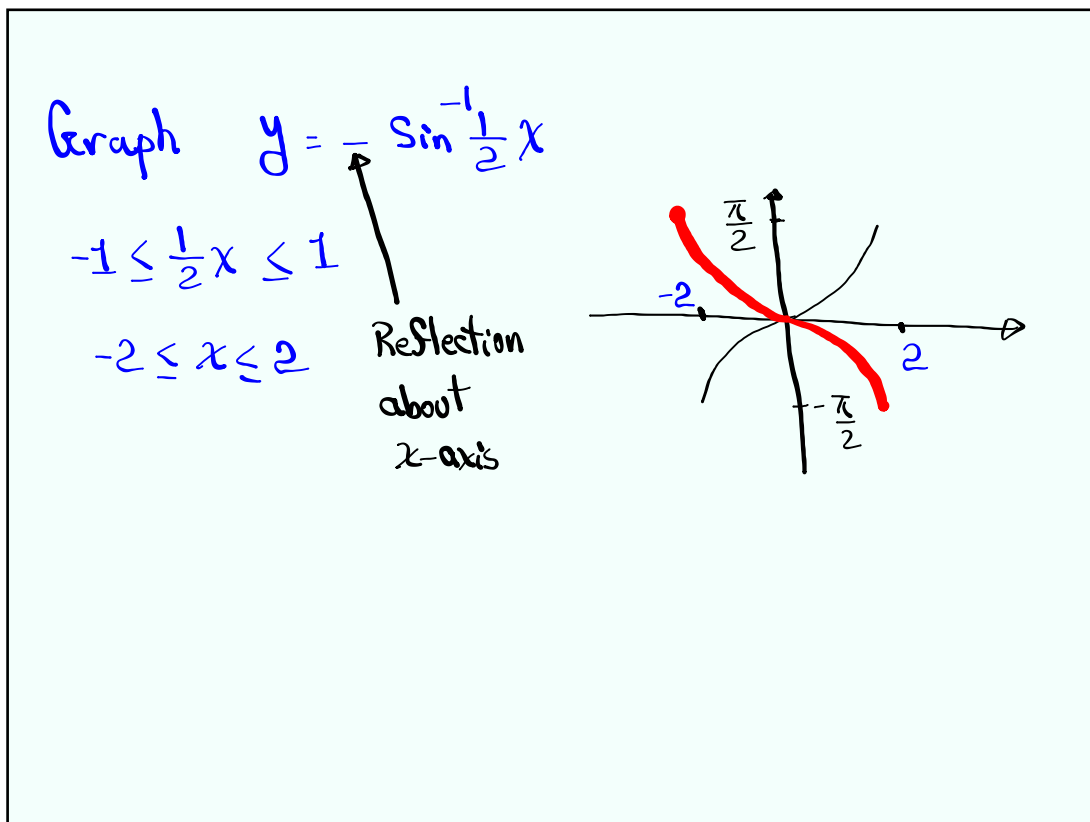
$$x^2 = 2^2 + 3^2 - 2 \cdot 2 \cdot 3 \cdot \cos 50^\circ$$

$$x^2 = 5.287 \quad \boxed{x \approx 2.3}$$

Dec 9-10:55 AM

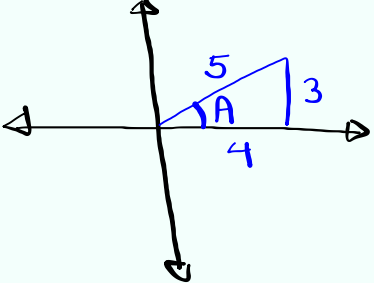


Dec 9-11:00 AM

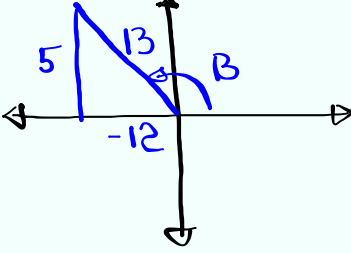


Dec 9-11:06 AM

$\sin A = \frac{3}{5}$ ,  $A$  is in QI



$\tan B = \frac{-5}{12}$ ,  $B$  is in QII.



$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

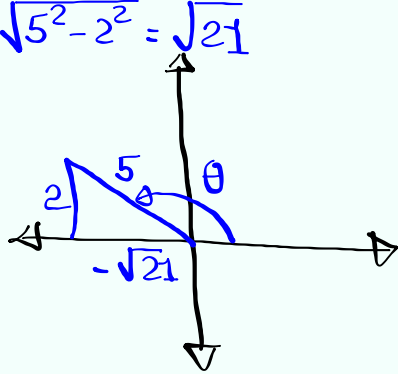
$$= \frac{3}{5} \cdot \frac{-12}{13} + \frac{4}{5} \cdot \frac{5}{13}$$

$$= \frac{-16}{65}$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A} = \frac{2 \cdot \frac{3}{4}}{1 - \left(\frac{3}{4}\right)^2} = \frac{\frac{3}{2}}{1 - \frac{9}{16}} = \frac{24}{7}$$
LCD=16

Dec 9-11:09 AM

$\sin \theta = \frac{2}{5}$ ,  $\theta$  is in QII



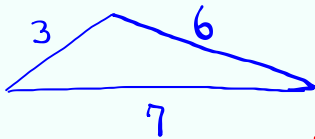
$$\sqrt{5^2 - 2^2} = \sqrt{21}$$

$\sin \theta = \frac{2}{5}$	$\csc \theta = \frac{5}{2}$
$\cos \theta = -\frac{\sqrt{21}}{5}$	$\sec \theta = -\frac{5}{\sqrt{21}}$
$\tan \theta = -\frac{2}{\sqrt{21}}$	$\cot \theta = -\frac{\sqrt{21}}{2}$

Rationalize  $\frac{-2}{\sqrt{21}} \cdot \frac{\sqrt{21}}{\sqrt{21}} = \frac{-2\sqrt{21}}{21}$

Dec 9-11:15 AM

find the area



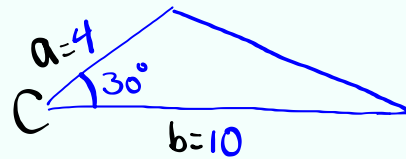
Heron's formula

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \frac{3+6+7}{2} = 8$$

$$= \sqrt{8(8-3)(8-7)(8-6)}$$

$$= \sqrt{8 \cdot 5 \cdot 1 \cdot 2} = \sqrt{80} \approx 9$$

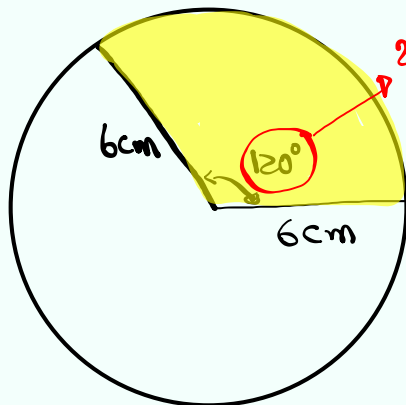


$$\text{Area} = \frac{1}{2} ab \sin C$$

$$= \frac{1}{2} \cdot 4 \cdot 10 \cdot \sin 30^\circ$$

$$= \frac{1}{2} \cdot 4 \cdot 10 \cdot \frac{1}{2} = 10 \text{ units}^2$$

Dec 9-11:20 AM



1) Find area of the Sector

$$A = \frac{1}{2} r^2 \theta = \frac{1}{2} (6)^2 \cdot \frac{2\pi}{3}$$

$$= \frac{36 \cdot 2\pi}{6} = 12\pi \text{ cm}^2$$

2) Find the arc length.

$$S = r\theta$$

$$= 6 \cdot \frac{2\pi}{3} = 4\pi \text{ cm}$$

Dec 9-11:25 AM

find all exact solution of

$$2\cos^2 x + \cos x - 1 = 0 \quad \text{over } [0, 2\pi).$$

$$(2\cos x - 1)(\cos x + 1) = 0$$

$$2\cos x - 1 = 0 \quad \text{OR} \quad \cos x + 1 = 0$$

$$\cos x = \frac{1}{2}$$

QI, QIV

$$\text{RA } \frac{\pi}{3}$$

$$\text{QI } x = \boxed{\frac{\pi}{3}}$$

$$\text{QIV } x = 2\pi - \frac{\pi}{3} = \boxed{\frac{5\pi}{3}}$$

$$\cos x = -1$$

$$\boxed{x = \pi}$$

$$\left\{ \frac{\pi}{3}, \pi, \frac{5\pi}{3} \right\}$$

Dec 9-11:30 AM

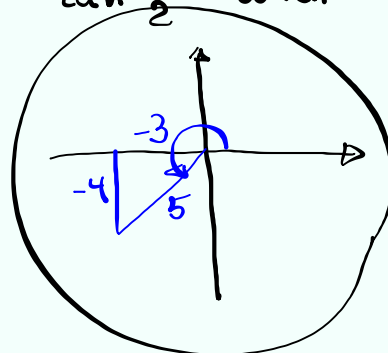
exact value of  $\tan \frac{x}{2}$  when  $\sin x = -\frac{4}{5}$

$$\tan \frac{x}{2} = \frac{\sin x}{1 + \cos x}$$

$$= \frac{-\frac{4}{5}}{1 + \frac{-3}{5}}$$

LCD=5

$$= \frac{-4}{5+(-3)} = \frac{-4}{2} = \boxed{-2}$$



$$\pi < x < \frac{3\pi}{2}$$

$$180^\circ < x < 270^\circ$$

$$90^\circ < \frac{x}{2} < 135^\circ$$

QII

$\tan$  is -

Dec 9-11:35 AM

$Z = 3 \text{cis} 15^\circ$       Find all  <sup>$n=3$</sup>  Cube roots  <sup>$3$  - Ans</sup> of  $64i$   
 $Z^4 = 3^4 \text{cis} 4 \cdot 15^\circ$        $64i = 0 + 64i$   
 $= \boxed{81 \text{cis} 60^\circ}$        $= 64 (\cos 90^\circ + i \sin 90^\circ)$

$$\sqrt[3]{64} \left[ \cos \left( \frac{90^\circ + k \cdot 360^\circ}{3} \right) + i \sin \left( \frac{90^\circ + k \cdot 360^\circ}{3} \right) \right]$$

$$= 4 \left[ \cos(30^\circ + k \cdot 120^\circ) + i \sin(30^\circ + k \cdot 120^\circ) \right]$$

$k=0 \rightarrow 4 [\cos 30^\circ + i \sin 30^\circ] = Z_1$   
 $k=1 \rightarrow 4 [\cos 150^\circ + i \sin 150^\circ] = Z_2$   
 $k=2 \rightarrow 4 [\cos 270^\circ + i \sin 270^\circ] = Z_3$

Dec 9-11:41 AM